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# Searching for $G^3$ in $t\bar{t}$ Production

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**Abstract**

The triple gluon field strength operator $G^3$ represents the only genuinely gluonic CP conserving term which can appear at dimension-6 within an effective strong interaction Lagrangian. Previous studies of this operator have revealed that its effect on gluon scattering is surprisingly difficult to detect. In this article, we analyze the impact of $G^3$ upon top quark pair production. We find that it will generate observable cross section deviations from QCD at the LHC for even relatively small values of its coefficient. Furthermore, $G^3$ affects the transverse momentum distribution of the produced top quarks more strongly at high energies than dimension-6 four-quark and chromomagnetic moment terms in the effective Lagrangian. Top-antitop production at the LHC will therefore provide a sensitive and clean probe for the elusive triple gluon field strength operator.

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## 1. Introduction

The quest to uncover signs of new physics beyond the Standard Model is being pursued across a broad front. One important area in which the search has been underway for many years is the strong interaction sector. Quantum chromodynamics has so far passed every experimental test and explains all strong interaction phenomenology at currently accessible energies. However, new color interactions beyond those of conventional QCD could emerge at higher energy scales from a number of different sources. For example, a wide range of theories beyond the Standard Model contain novel particles which couple to quarks and gluons. Such particles could be squarks and gluinos in supersymmetric theories [1], colored technihadrons in non-minimal technicolor models [2], or fermions that transform according to representations of color $SU(3)$ other than the fundamental triplet in certain electroweak symmetry breaking scenarios [3]. New strong sector physics might alternatively originate from quark or gluon substructure [4]. Preon exchange between composite states would generally induce nonstandard quark or gluon couplings. In short, the variety of novel color interactions which have been proposed in the literature provides many possible sources of interesting physics.

A complete description of any new strong interaction phenomena requires a fundamental theory beyond the Standard Model. However at energies below its characteristic scale $\Lambda$, the new physics may be studied within an effective field theory framework. Its low energy effects can be reproduced by supplementing the renormalizable terms in the QCD Lagrangian with higher dimension operators. The resulting effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{QCD} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)}(\mu) O_i^{(6)}(\mu) + \frac{1}{\Lambda^4}\sum_i C_i^{(8)}(\mu) O_i^{(8)}(\mu) + O\big(\frac{1}{\Lambda^6}\big) \tag{1.1}$$

generally contains all terms consistent with $SU(3)\times SU(2)\times U(1)$ gauge invariance and any imposed discrete or global symmetries. The operators $O_i^{(n)}$ and their dimensionless coefficients $C_i^{(n)}$ are grouped together in eqn. (1.1) according to their mass dimension $n$ and multiplied by a factor of $1/\Lambda^{n-4}$ so that their overall combined dimension equals 4. As a result, the most important nonrenormalizable terms in $\mathcal{L}_{\rm eff}$ are those of dimension-6 since they are least suppressed by inverse powers of the high energy scale $\Lambda$.

In the quark sector, one can form a large number of dimension-6 operators with different chiral and color structures [5,6]. The qualitative impact of these terms upon quark scattering has conventionally been assessed by including just one representative four-quark operator such as

$$O^{(6)}_{4\text{-quark}} = \frac{1}{2}(\overline{q}_L\gamma^\mu q_L)(\overline{q}_L\gamma_\mu q_L) \tag{1.2}$$

into the effective Lagrangian and setting the magnitude of its coefficient equal to $4\pi$. The scale $\Lambda$ associated with the four-quark operator has then been constrained by comparing its effect upon the theoretical prediction for the inclusive jet cross section with experimental measurement. A recent such comparison yields the bound $\Lambda > 1.4$ TeV [7].

The number of nonrenormalizable terms which arise at dimension-6 in the gluon sector is much more limited. One can build only two gauge invariant operators out of covariant derivatives $D^\mu = \partial^\mu - ig_s G^\mu_a T_a$ and gluon field strengths $G^{\mu\nu}_a = \partial^\mu G^\nu_a - \partial^\nu G^\mu_a + g_s f_{abc} G^\mu_b G^\nu_c$ that preserve $C$, $P$ and $T$ [8]:

$$O^{(6)}_1 = g_s f_{abc} G^\mu_{a\nu} G^\nu_{b\lambda} G^\lambda_{c\mu} \tag{1.3$a$}$$

$$O^{(6)}_2 = \frac{1}{2} D^\mu G^a_{\mu\nu} D_\lambda G^{\lambda\nu}_a. \tag{1.3$b$}$$

Other candidates such as $g_s d_{abc} G^\mu_{a\nu} G^\nu_{b\lambda} G^\lambda_{c\mu}$ or $\frac{1}{2} D^\lambda G^a_{\mu\nu} D_\lambda G^{\mu\nu}_a$ either vanish or reduce to combinations of $O^{(6)}_1$ and $O^{(6)}_2$. The triple gluon field strength term in (1.3$a$), which we shall name $G^3$ for short, represents a true gluonic operator. It contributes for instance to the distinctive three-point vertex in the nonabelian color theory. On the other hand, the double gluon field strength operator in (1.3$b$), which we will call $(DG)^2$, is not really gluonic. The classical equation of motion

$$D_\mu G^{\mu\nu}_a = -g_s \sum_{\text{flavors}} \overline{q}\gamma^\nu T_a q \tag{1.4}$$

relates its S-matrix elements to those of a color octet four-quark operator [9]:

$$O^{(6)}_2 \xrightarrow{EOM} \frac{g_s^2}{2} \sum_{\text{flavors}} \big(\overline{q}\gamma_\mu T_a q\big)\big(\overline{q}\gamma^\mu T_a q\big). \tag{1.5}$$

The double field strength operator thus affects parton processes involving external quarks rather than external gluons. While the origin of $O^{(6)}_2$ could significantly differ from those of other four-quark operators, its impact upon parton scattering is not very different from theirs. So at dimension-6, only one genuinely gluonic term which conserves $CP$ may appear within the effective Lagrangian.

The list of $CP$ even gluon operators grows at dimension-8 but remains manageable in size. It is useful to classify them according to the number of field strengths that they contain [10]. Only one independent operator can be built out of two field strengths and four covariant derivatives:

$$O_1^{(8)} = \frac{1}{2} D^\mu G^a_{\mu\nu} D^2 D_\lambda G_a^{\lambda\nu}. \tag{1.6$a$}$$

Any other candidate in this category such as $\frac{1}{2} D^2 G^a_{\mu\nu} D^2 G_a^{\mu\nu}$ may be reduced via the Jacobi identity $D_\lambda G^a_{\mu\nu} + D_\mu G^a_{\nu\lambda} + D_\nu G^a_{\lambda\mu} = 0$ to $O_1^{(8)}$ plus operators with more than two field strengths. There are two possibilities for dimension-8 operators with three gluon field strengths and two covariant derivatives: [1]

$$\begin{aligned} O_2^{(8)} &= g_s f_{abc} G^\mu_{a\nu} D_\lambda G_b^{\lambda\nu} D^\sigma G^c_{\sigma\mu} \\ O_3^{(8)} &= g_s f_{abc} G^\mu_{a\nu} G^\nu_{b\lambda} D^2 G^\lambda_{c\mu}. \end{aligned} \tag{1.6$b$}$$

Finally, six independent operators containing four field strengths can be formed [11]:

$$\begin{aligned} O_4^{(8)} &= \frac{g_s^2}{2} G^a_{\mu\nu} G_a^{\mu\nu} G^b_{\lambda\sigma} G_b^{\lambda\sigma} \\ O_5^{(8)} &= \frac{g_s^2}{2} G^a_{\mu\nu} \tilde{G}_a^{\mu\nu} G^b_{\lambda\sigma} \tilde{G}_b^{\lambda\sigma} \\ O_6^{(8)} &= \frac{g_s^2}{2} G^a_{\mu\nu} G_b^{\mu\nu} G^a_{\lambda\sigma} G_b^{\lambda\sigma} \\ O_7^{(8)} &= \frac{g_s^2}{2} G^a_{\mu\nu} \tilde{G}_b^{\mu\nu} G^a_{\lambda\sigma} \tilde{G}_b^{\lambda\sigma} \\ O_8^{(8)} &= \frac{g_s^2}{2} d_{abe} d_{cde} G^a_{\mu\nu} G_b^{\mu\nu} G^c_{\lambda\sigma} G_d^{\lambda\sigma} \\ O_9^{(8)} &= \frac{g_s^2}{2} d_{abe} d_{cde} G^a_{\mu\nu} \tilde{G}_b^{\mu\nu} G^c_{\lambda\sigma} \tilde{G}_d^{\lambda\sigma}. \end{aligned} \tag{1.6$c$}$$

The equation of motion in (1.4) relates the S-matrix elements of $O_1^{(8)}$ and $O_2^{(8)}$ to those of operators with four quark fields. So among all the dimension-8 operators, only $O_3^{(8)}$ - $O_9^{(8)}$ are genuinely gluonic and affect scattering processes with external gluons.

Present day experiments at the Fermilab Tevatron are more sensitive to the presence of nonrenormalizable quark operators in the strong interaction effective Lagrangian than

[1] Operator $O_3^{(8)}$ does not reduce to $O_2^{(8)}$ plus a four field strength operator as previously reported in ref. [10].

to their gluonic counterparts. But at higher energy machines such as the LHC, the gluon content of colliding hadrons at small parton momenta fractions dominates over that of valence and sea quarks. Future collider experiments will therefore probe possible deviations from the Standard Model much more sensitively in the gluon rather than quark sectors. In this article, we will investigate the prospects for detecting such deviations. In particular, we will focus upon searching for the triple gluon field strength operator $G^3$.

The most obvious channel in which to look for $G^3$ might seem to be $gg \to gg$ scattering. However, the helicity structure of the amplitude for this process which involves the gluon operator is orthogonal to that of pure QCD. They consequently do not interfere at $O(1/\Lambda^2)$ [8]. At $O(1/\Lambda^4)$, the dimension-8 operators $O_3^{(8)}$ - $O_9^{(8)}$ enter into the $gg \to gg$ amplitude and contaminate any signal from $O_1^{(6)}$ [10]. So looking for $G^3$ in lowest order gluon scattering is difficult. As we shall later see, the triple field strength operator does not mix at one-loop order with any four-quark operators. Its presence in the effective Lagrangian thus cannot be indirectly inferred from pure quark processes either.

These obstacles to observing $G^3$ have motivated consideration of other possible detection methods. For instance, it may be probed in $Z \to 4$ jets decay at LEP [12,13]. This option is limited, though, by LEP's relatively low energy. Alternatively, one can look for the effect of $G^3$ upon $gg \to ggg$ scattering in three-jet events [14]. The task of identifying its signature in this next-to-leading order channel represents a formidable experimental challenge. The triple gluon field strength operator has thus remained stubbornly difficult to detect.

In this paper, we take a new approach to searching for the $G^3$ operator which possesses significant advantages over those previously explored. Specifically, we investigate the effect of the triple field strength term upon top quark pair production. As we shall demonstrate, the impact of $G^3$ at LHC energies is sizable for even relatively small values of its effective Lagrangian coefficient. It dominates in this mode over four-quark competitors with comparable coefficients. Moreover, contamination from gluon terms of dimension-8 and higher is small. So $t\bar{t}$ production provides a sensitive and clean probe for the elusive $G^3$ operator.

Our article is organized as follows. In section 2, we discuss the renormalization group running of all the dimension-6 operator coefficients that influence top quark pair production. We then compute in section 3 the $t\bar{t}$ differential transverse momentum cross sections at the LHC and Tevatron for representative values of their operator coefficients and compare with corresponding QCD results. In section 4, we quantify the difference between the transverse momentum cross sections calculated with and without the gluonic operators in the effective Lagrangian as a function of their coefficients. In section 5, we investigate the operators' impact upon $t\bar{t}$ angular distributions. Finally, we summarize our findings and present our conclusions in section 6.

## 2. Dimension-6 operator coefficients

Many forms of physics beyond the Standard Model could give rise to nonrenormalizable gluonic operators within the effective strong interaction Lagrangian. In the absence of a fundamental theory, the coefficients of such operators are *a priori* unknown. Even when a particular high energy model is specified, their values may prove intractably difficult to calculate. So in this work, we will simply study the feasibility of detecting the lowest dimension operators as a function of their coefficient values.

Since the dimensionless coefficients and dimensionful scale appear together within the effective Lagrangian (1.1), they cannot be distinguished at tree level. We are therefore free to fix the value for the high energy scale. Motivated by the expectation that a new fundamental layer of physics awaits discovery in the TeV regime, we will take it to be $\Lambda = 2$ TeV. This relatively large value for the effective theory cutoff ensures that our low energy analysis will be valid over almost the entire practical energy range of present and anticipated hadron colliders.

Although we will generally treat the coefficients of the gluon operators as free parameters, we note that there is an important special case in which they can readily be computed. This occurs when the operators are generated through diagrams involving heavy colored particle loops like those illustrated in fig. 1. At energies below the particle's mass, the behavior of such graphs may be reexpressed in terms of local but nonrenormalizable gluonic operators. The values for their coefficients are then determined by performing a matching computation. For example, if the particles running around the loop in fig. 1 are fermions of mass $\Lambda$ and belong to representation $R$ of color $SU(3)$, the sum of the diagrams contains terms which match onto the dimension-6 operators in eqn. (1.3) with coefficients

$$\begin{aligned} C_1^{(6)}(\Lambda) &= -\frac{K(R)}{360}\frac{\alpha_s(\Lambda)}{\pi} \\ C_2^{(6)}(\Lambda) &= -\frac{K(R)}{15}\frac{\alpha_s(\Lambda)}{\pi} \end{aligned} \tag{2.1}$$

where $K(R) = \sum_{a=1}^{8} \mathrm{Tr}[T_a(R)T_a(R)]/8$ denotes the index of $R$. The corresponding result for colored scalars is given by

$$\begin{aligned} C_1^{(6)}(\Lambda) &= \frac{K(R)}{720}\frac{\alpha_s(\Lambda)}{\pi} \\ C_2^{(6)}(\Lambda) &= -\frac{K(R)}{120}\frac{\alpha_s(\Lambda)}{\pi}. \end{aligned} \tag{2.2}$$

Unfortunately, the numerical values of these coefficients are too small in any reasonable theory to produce detectable deviations from the Standard Model. We will therefore have to be content with constraining mechanisms other than perturbative radiative corrections which can generate gluonic operators with larger coefficients.

Whatever values $C_1^{(6)}$ and $C_2^{(6)}$ may assume at the scale $\Lambda$, they must be evolved down to the energies at which $O_1^{(6)}$ and $O_2^{(6)}$ are probed using the renormalization group. We consequently need to enlarge our operator basis so that it closes under renormalization. It is sensible to apply the equations of motion to reduce the operator set as much as possible. We then find that $O_1^{(6)}$ and $O_2^{(6)}$ do not mix with each other under the action of QCD at one-loop order. Instead, $O_1^{(6)}$ runs into itself and the chromomagnetic moment operator

$$O_0^{(6)} = \sum_{\text{flavors}} g_s m_q \bar{q}\sigma^{\mu\nu}T^a q G^a_{\mu\nu}. \tag{2.3}$$

Their evolution is governed by the $2\times 2$ anomalous dimension matrix

$$\gamma_I = \begin{array}{c} \\ O_0^{(6)} \\ O_1^{(6)} \end{array} \!\!\begin{array}{c} \begin{array}{cc} \quad O_0^{(6)} & \qquad O_1^{(6)} \end{array} \\ \begin{pmatrix} 14/3 & 0 \\ 9/2 & 7+2n_f/3 \end{pmatrix} \end{array} \frac{g_s^2}{8\pi^2} + O(g_s^4) \tag{2.4}$$

where $n_f$ denotes the number of active quark flavors [11,15]. In the $O_2^{(6)}$ sector, the equations of motion convert the double gluon field strength operator into the color octet four-quark term in (1.5). $O_2^{(6)}$ then mixes at one-loop order with its four-quark counterparts

$$\begin{aligned} O_3^{(6)} &= \frac{g_s^2}{2}\sum_{\text{flavors}} \left(\bar{q}\gamma_\mu\gamma^5 T_a q\right)\left(\bar{q}\gamma^\mu\gamma^5 T_a q\right) \\ O_4^{(6)} &= \frac{g_s^2}{2}\sum_{\text{flavors}} \left(\bar{q}\gamma_\mu q\right)\left(\bar{q}\gamma^\mu q\right) \\ O_5^{(6)} &= \frac{g_s^2}{2}\sum_{\text{flavors}} \left(\bar{q}\gamma_\mu\gamma^5 q\right)\left(\bar{q}\gamma^\mu\gamma^5 q\right) \end{aligned} \tag{2.5}$$

through the $4 \times 4$ matrix

$$\gamma_{II} = \begin{array}{c} \\ O_2^{(6)} \\ O_3^{(6)} \\ O_4^{(6)} \\ O_5^{(6)} \end{array} \begin{array}{c} \begin{array}{cccc} O_2^{(6)} & O_3^{(6)} & O_4^{(6)} & O_5^{(6)} \end{array} \\ \begin{pmatrix} 311/36 - 2n_f/3 & 5/4 & 0 & 2/3 \\ 41/36 & 35/4 - 2n_f/3 & 2/3 & 0 \\ 4/3 & 6 & 11 - 2n_f/3 & 0 \\ 22/3 & 0 & 0 & 11 - 2n_f/3 \end{pmatrix} \end{array} \frac{g_s^2}{8\pi^2} + O(g_s^4). \tag{2.6}$$

The coefficients of operators $O_0^{(6)}$ - $O_5^{(6)}$ satisfy the integrated renormalization group equation

$$C_i(\mu) = \sum_j \Big[\exp \int_{g_s(\Lambda)}^{g_s(\mu)} dg_s \frac{\gamma^T(g_s)}{\beta(g_s)}\Big]_{ij} C_j(\Lambda) \tag{2.7}$$

where $\beta(g_s)$ denotes the QCD beta function. [2] The leading log running of these coefficients can be substantial. For example, if they assume the values

$$(C_0^{(6)}, C_1^{(6)}, C_2^{(6)}, C_3^{(6)}, C_4^{(6)}, C_5^{(6)})(\Lambda) = (1, 1, 1, 0, 0, 0) \tag{2.8}$$

at the canonical $\Lambda = 2$ TeV scale, the coefficients run down to

$$\begin{aligned} &(C_0^{(6)}, C_1^{(6)}, C_2^{(6)}, C_3^{(6)}, C_4^{(6)}, C_5^{(6)})(2m_t) = \\ &\qquad (0.7858, 0.7458, 0.8856, -0.0294, 0.0003, -0.0152) \end{aligned} \tag{2.9}$$

at the top-antitop threshold. [3] Renormalization group evolution therefore generally suppresses the operator coefficients.

## 3. Top quark pair production

The sensitivity of a hadron collider experiment to nonrenormalizable terms in the effective strong interaction Lagrangian is a function of the accelerator's center-of-mass

[2] We take the constant of integration which enters into the integrated QCD beta function to be $\alpha_s(M_z) = 0.125$ [17]. The corresponding value for the strong interaction fine structure constant at $\Lambda = 2$ TeV is $\alpha_s(\Lambda) = 0.086$ which implies $g_s(\Lambda) = 1.04$. Since this coupling is very close to unity, the value of $C_1^{(6)}(\Lambda)$ is essentially independent of the number of powers of $g_s$ that are included into the $G^3$ operator's definition.

[3] We adopt the recently reported CDF central value $m_t = 174$ GeV for the mass of the top quark [16].

energy $\sqrt{s}$ and luminosity $\mathcal{L}$. The Fermilab Tevatron currently operates at $\sqrt{s} = 1.8$ TeV and has collected approximately 20 $\mathrm{pb}^{-1}$ of data. These values for $\sqrt{s}$ and $\int \mathcal{L} dt$ are too low to allow for any significant limit to be placed upon the coefficient of the triple gluon field strength operator $G^3$. However, the prospects for probing $G^3$ will substantially improve with the advent of the LHC. This machine is projected to run at $\sqrt{s} = 14$ TeV and collect approximately 30 $\mathrm{fb}^{-1}$ of data per year. At such high energies, the gluon content of the colliding hadrons at small parton momenta fractions dominates over that of all other partons. Sensitivity to the triple field strength operator will consequently be greatly enhanced.

As mentioned in the introduction, looking for $G^3$ in $gg \to gg$ scattering is problematic. The only other $2 \to 2$ parton process that maximally benefits from the large numbers of gluons in a high $\sqrt{s}$ initial state is $gg \to q\overline{q}$. The $O(1/\Lambda^2)$ interference between the QCD and $G^3$ amplitudes for this mode is proportional to the squared quark mass ${m_q}^2$. It is therefore greatest when the final state quark flavor is top. The top quark is also the easiest to tag since its leptonic decay channel can produce a high energy, isolated lepton in conjunction with a bottom quark. This distinctive signature cuts down on genuine backgrounds as well as false identifications. If in the future $b\overline{b}$ and even $c\overline{c}$ final states can be positively identified, then the signal for $G^3$ will only be enhanced and all our results can be easily applied to their study. But for now, we shall examine the impact of the nonrenormalizable terms in $\mathcal{L}_{\mathrm{eff}}$ upon just top quark pair production. As we shall see, this mode provides a sensitive and clean probe for the $G^3$ operator.

Top quark pair production proceeds at tree level through the parton reactions $gg \to t\overline{t}$ and $q\overline{q} \to t\overline{t}$. We first consider the gluon channel. The lowest order QCD graphs that mediate $gg \to t\overline{t}$ scattering are illustrated in fig. 2a. At $O(1/\Lambda^2)$, the chromomagnetic moment and triple gluon field strength operators $O_0^{(6)}$ and $O_1^{(6)}$ contribute through the diagrams shown in fig. 2b and fig. 2c. They also enter into the $gg \to t\overline{t}$ amplitude at $O(1/\Lambda^4)$ via the double operator insertion graphs displayed in fig. 2d. The only other possible dimension-6 operator tree diagrams contain insertions of the double field strength operator $O_2^{(6)}$. But the sum of such $(DG)^2$ graphs vanishes as guaranteed by the S-matrix relation in (1.5).

After adding together all the diagrams in fig. 2 and squaring the total $gg \to t\bar{t}$ amplitude, we find [4]

$$
\begin{aligned}
\overline{\sum}' |\mathcal{A}(gg \to t\bar{t})|^2 = {} & \frac{3}{4}\frac{(m_t^2-\hat{t})(m_t^2-\hat{u})}{\hat{s}^2} - \frac{1}{24}\frac{m_t^2(\hat{s}-4m_t^2)}{(m_t^2-\hat{t})(m_t^2-\hat{u})} \\
& + \frac{1}{6}\Big[\frac{\hat{t}\hat{u}-m_t^2(3\hat{t}+\hat{u})-{m_t}^4}{(m_t^2-\hat{t})^2} + \frac{\hat{t}\hat{u}-m_t^2(\hat{t}+3\hat{u})-{m_t}^4}{(m_t^2-\hat{u})^2}\Big] \\
& - \frac{3}{8}\Big[\frac{\hat{t}\hat{u}-2m_t^2\hat{t}+{m_t}^4}{\hat{s}(m_t^2-\hat{t})} + \frac{\hat{t}\hat{u}-2m_t^2\hat{u}+{m_t}^4}{\hat{s}(m_t^2-\hat{u})}\Big] \\
& + \frac{1}{\Lambda^2}\Big[\frac{1}{3}C_0^{(6)}\frac{m_t^2(4\hat{s}^2-9\hat{t}\hat{u}-9m_t^2\hat{s}+9{m_t}^4)}{(m_t^2-\hat{t})(m_t^2-\hat{u})} \\
& \quad + \frac{9}{8}C_1^{(6)}\frac{m_t^2(\hat{t}-\hat{u})^2}{(m_t^2-\hat{t})(m_t^2-\hat{u})}\Big] \\
& + \frac{1}{\Lambda^4}\Big[\frac{1}{6}{C_0^{(6)}}^2\frac{m_t^2\big(14\hat{s}\hat{t}\hat{u}+m_t^2(31\hat{s}^2-36\hat{t}\hat{u})-50{m_t}^4\hat{s}+36{m_t}^6\big)}{(m_t^2-\hat{t})(m_t^2-\hat{u})} \\
& \quad + \frac{9}{8}C_0^{(6)}C_1^{(6)}\frac{m_t^2\hat{s}^3}{(m_t^2-\hat{t})(m_t^2-\hat{u})} + \frac{27}{4}{C_1^{(6)}}^2(m_t^2-\hat{t})(m_t^2-\hat{u})\Big] + O\Big(\frac{1}{\Lambda^6}\Big).
\end{aligned}
\tag{3.1}
$$

The bar appearing over the summation symbol on the LHS of (3.1) implies that the squared matrix element is averaged (summed) over initial (final) spins and colors, while the prime indicates that $|A(gg \to t\bar{t})|^2$ is divided by $g_s^4$. Notice that all of the nonrenormalizable operator terms except the last one are proportional to $m_t^2$. The corresponding terms in the squared $gg \to q\bar{q}$ amplitude for quark flavors other than top are therefore negligible by comparison. The last term in (3.1) is significantly enhanced by its 27/4 prefactor. Moreover, it increases quadratically in the partonic Mandelstam invariants $\hat{s}$, $\hat{t}$ and $\hat{u}$. In contrast, the other $O(1/\Lambda^4)$ terms only grow linearly, while the $O(1/\Lambda^2)$ terms approach a constant. So away from the $t\bar{t}$ threshold and over large regions of $C_1^{(6)}$ parameter space, the $O_1^{(6)}$ operator's squared amplitude is much larger than its interference with QCD. As a result, the impact of $G^3$ upon $t\bar{t}$ production depends mainly upon the magnitude rather than sign of its coefficient.

Since the gluonic term proportional to ${C_1^{(6)}}^2$ in (3.1) is surprisingly large, one may question whether other $O(1/\Lambda^4)$ terms arising from dimension-8 gluon operators could

[4] The results displayed here in eqns. (3.1) and (3.3) correct errors in eqns. (14a) and (14b) of ref. [18] and eqns. (4.4) and (4.5) of ref. [10]. The pure QCD terms in these formulae agree with the results of ref. [19].

be significant as well. The answer is generally no. Recall that among the dimension-8 operators listed in eqn. (1.6), $O_1^{(8)}$ and $O_2^{(8)}$ enter at tree level into processes involving at least four quarks, while vertices from $O_4^{(8)}$ - $O_9^{(8)}$ contain at least four gluons. So only $O_3^{(8)}$ can affect $gg \to t\bar{t}$ scattering at lowest order. The interference between its amplitude and that of pure QCD yields the $O(1/\Lambda^4)$ term

$$\overline{\sum}' |\mathcal{A}(gg \to t\bar{t})|^2 = \cdots - \frac{3}{8} \frac{C_3^{(8)}}{\Lambda^4} \frac{m_t^2 \hat{s} (\hat{t} - \hat{u})^2}{(m_t^2 - \hat{t})(m_t^2 - \hat{u})}. \tag{3.2}$$

This dimension-8 term has a much smaller prefactor and increases more slowly with $\hat{s}$, $\hat{t}$ and $\hat{u}$ than its dimension-6 competitor. So unless its coefficient $C_3^{(8)}$ is more than an order of magnitude larger than $C_1^{(6)}$, the $O_3^{(8)}$ operator is not likely to obscure any signal from $O_1^{(6)}$.

We now turn to consider the quark process $q\bar{q} \to t\bar{t}$. The chromomagnetic moment, double gluon field strength and four-quark operators in our basis contribute at lowest order in the strong interaction coupling to this channel as indicated in fig. 3. Neglecting all quark masses except that of the top in the amplitude sum, we find

$$\begin{aligned}
\overline{\sum}' |\mathcal{A}(q\bar{q} \to t\bar{t})|^2 = & \frac{4}{9\hat{s}^2} \big[\hat{t}^2 + \hat{u}^2 + 4m_t^2 \hat{s} - 2{m_t}^4\big] \\
& + \frac{8}{9\hat{s}\Lambda^2} \big[4C_0^{(6)} m_t^2 \hat{s} + C_2^{(6)} (\hat{t}^2 + \hat{u}^2 + 4m_t^2\hat{s} - 2{m_t}^4) + C_3^{(6)} \hat{s} (\hat{t} - \hat{u})\big] \\
& + \frac{4}{9\Lambda^4} \Big[8{C_0^{(6)}}^2 m_t^2 (\hat{t}\hat{u} + 2m_t^2 \hat{s} - {m_t}^4)/\hat{s} \\
& \qquad + 8C_0^{(6)} C_2^{(6)} m_t^2 \hat{s} + 8C_0^{(6)} C_3^{(6)} m_t^2 (\hat{t} - \hat{u}) \\
& \qquad + ({C_2^{(6)}}^2 + \frac{1}{2}{C_4^{(6)}}^2)(\hat{t}^2 + \hat{u}^2 + 4m_t^2 \hat{s} - 2{m_t}^4) \\
& \qquad + ({C_3^{(6)}}^2 + \frac{1}{2}{C_5^{(6)}}^2)(\hat{t}^2 + \hat{u}^2 - 2{m_t}^4) \\
& \qquad + (2C_2^{(6)} C_3^{(6)} + C_4^{(6)} C_5^{(6)}) \hat{s} (\hat{t} - \hat{u})\Big].
\end{aligned} \tag{3.3}$$

Unlike the gluonic scattering result in (3.1), this expression contains no anomalously large $O(1/\Lambda^4)$ term. So we expect that the effect of dimension-8 and higher operators upon $q\bar{q} \to t\bar{t}$ scattering is small.

The squared amplitudes in (3.1) and (3.3) enter into the partonic differential cross section

$$\frac{d\sigma(ab \to t\bar{t})}{d\hat{t}} = \frac{\pi \alpha_s^2}{\hat{s}^2} \overline{\sum}' |\mathcal{A}(ab \to t\bar{t})|^2 \tag{3.4}$$

which appears in the full hadronic differential cross section for top-antitop production [20]:

$$\frac{d^3\sigma}{dy_3 dy_4 dp_\perp}(AB \to t\bar{t}) = 2p_\perp \sum_{ab} x_a x_b f_{a/A}(x_a) f_{b/B}(x_b) \frac{d\sigma(ab \to t\bar{t})}{d\hat{t}}. \tag{3.5}$$

The partonic cross section is folded together with distribution functions $f_{a/A}(x_a)$ and $f_{b/B}(x_b)$ that specify the probability of finding partons $a$ and $b$ inside hadrons $A$ and $B$ carrying momentum fractions $x_a$ and $x_b$. The product is then summed over initial parton configurations. The resulting hadronic cross section is a function of the top and antitop rapidities $y_3$ and $y_4$ and their common transverse momentum $p_\perp$.

The triply differential cross section in (3.5) may be reduced to a function of a single variable by integrating over two of its independent degrees of freedom. We will concentrate upon the transverse momentum differential cross section which we obtain by integrating $d^3\sigma/dy_3 dy_4 dp_\perp$ over the rapidity range $-2.5 \le y_3, y_4 \le 2.5$. [5] The resulting $p_\perp$ distribution of $t\bar{t}$ pairs produced at the LHC is plotted in fig. 4. [6] The solid curve in the figure illustrates the QCD differential cross section $d\sigma_{QCD}(PP \to t\bar{t})/dp_\perp$. The dot-dashed, dashed and dotted curves delineate the contributions from operators $O_0^{(6)}$, $O_1^{(6)}$ and $O_2^{(6)}$ that are generated after respectively setting $C_0^{(6)}(\Lambda) = 0.5$, $C_1^{(6)}(\Lambda) = 0.5$ and $C_2^{(6)}(\Lambda) = 0.5$ in $\mathcal{L}_{\rm eff}$ with $\Lambda = 2$ TeV. The QCD and nonrenormalizable operator curves must be added together to obtain the effective field theory differential cross sections $d\sigma_{EFT}(PP \to t\bar{t})/dp_\perp$ that correspond to these nonzero coefficient values.

Several points about the results displayed in fig. 4 should be noted. Firstly, our choice for the operator coefficients is simply representative. Larger coefficient values lead to greater differences between the QCD and EFT curves, while smaller values diminish the discrepancies. Our particular choice for these parameters and the scale $\Lambda$ yields the combined coefficient $C_{0,1,2}^{(6)}(\Lambda)/\Lambda^2 = 0.5/(2 \text{ TeV})^2 \simeq 4\pi/(10 \text{ TeV})^2$ for the dimension-6 operators in effective Lagrangian (1.1). This value for the total coefficient is quite conservative compared to that which has typically been used in previous quark substructure

[5] This rapidity range does not represent a fiducial cut but rather a reasonable integration interval which contains the bulk of the produced top quarks. We have checked that extending the range to $-6 \le y_3, y_4 \le 6$ does not noticibly alter our final results.

[6] All of the results in fig. 4 and subsequent figures were calculated using the next-to-leading order parton distribution function set B of Harriman, Martin, Roberts and Stirling [21] evaluated at the renormalization scale $\mu = m_\perp \equiv \sqrt{m_t^2 + {p_\perp}^2}$.

studies. Secondly, the transverse momentum dependence of the curves in fig. 4 differentiates the dimension-6 operators from each other and the QCD terms in $\mathcal{L}_{\rm eff}$. At low values of $p_\perp$, the contribution to $d\sigma(PP \to t\bar{t})/dp_\perp$ from the magnetic moment operator $O_0^{(6)}$ dominates over those from the triple and double gluon field strength operators $O_1^{(6)}$ and $O_2^{(6)}$. But the $O_0^{(6)}$ curve falls off much more rapidly with increasing transverse momentum. So by placing a lower $p_\perp$ cut around 500 GeV, one can eliminate most of the chromomagnetic moment operator's contribution while retaining much of that from the double and triple field strength operators. Finally, all the curves in the figure will be shifted around by higher order QCD corrections. The next-to-leading $O(\alpha_s^3)$ corrections to the tree level QCD differential cross section may be comparable to or even larger than the $O(\alpha_s^2)$ deviations induced by the dimension-6 terms in $\mathcal{L}_{\rm eff}$ depending upon their coefficients. But the QCD and EFT distributions should be compared at the same order in $\alpha_s$. Only then can deviations between them be attributed to new physics beyond the Standard Model.

It is interesting to compare the LHC differential cross sections in fig. 4 with their Tevatron analogues shown in fig. 5. The Tevatron curves were calculated at $\sqrt{s} = 1.8$ TeV using the same values for the nonrenormalizable operator coefficients. Not surprisingly, the total integrated cross-section for $t\bar{t}$ production is two orders of magnitude lower at the Tevatron than at the LHC. Event rate is thus more of an issue at the lower-energy machine. We also clearly see from the two figures that the relative importance of the dimension-6 terms in the effective Lagrangian depends upon collider center-of-mass energy. At the Tevatron, $O_2^{(6)}$ dominates over $O_1^{(6)}$ for equal values of their high energy scale coefficients. This finding is intuitively reasonable since the gluon content of colliding hadrons at $\sqrt{s} = 1.8$ TeV is less important than at $\sqrt{s} = 14$ TeV.

The double gluon field strength operator affects quark scattering for all flavors. By comparing its predicted impact upon the inclusive jet cross section with 1988 Tevatron data, we have previously set an upper bound $|C_2^{(6)}(\Lambda)|/\Lambda^2 \le 4\pi/(2\ \text{TeV})^2$ on its coefficient [18]. If $C_2^{(6)}$ is allowed to assume this limiting value, we find that $O_2^{(6)}$ doubles the total integrated top quark cross section relative to the lowest order QCD prediction. This result is intriguing in light of the recent CDF measurement $\sigma(P\overline{P} \to t\bar{t})_{\rm CDF}/\sigma(P\overline{P} \to t\bar{t})_{QCD} = (13.9^{+6.1}_{-4.8}\,\text{pb})/(5.10^{+0.73}_{-0.43}\,\text{pb}) \simeq 2.7^{+1.2}_{-1.0}$ [16,22]. While it is premature to draw any conclusion from this observation, we believe it is safe to say that the bound on $C_2^{(6)}$ should be significantly strengthened in the future by Tevatron top quark data.

## 4. Mapping the coefficient parameter space

In our effective field theory framework, the low energy effects of any new strong interaction physics are encoded into the coefficients of the nonrenormalizable terms in the effective Lagrangian. These coefficients define a multi-dimensional parameter space. In order to assess the likelihood of detecting signals of new strong sector physics at the LHC, we need to map this space and determine the regions where deviations from QCD could be measured. Within those regions, we would then like to know whether it is possible to isolate effects from individual operators in $\mathcal{L}_{\text{eff}}$. We will explore these issues for the dimension-6 operators in our basis in this section.

To begin, we need to identify a measure of the difference between the predictions of QCD and the strong interaction effective theory for top quark pair production. We will focus upon the disparities in their LHC transverse momentum differential cross sections $d\sigma_{QCD}(PP \to t\bar{t})/dp_\perp$ and $d\sigma_{EFT}(PP \to t\bar{t})/dp_\perp$. It is important to recall that experimental systematic errors will inevitably render uncertain the absolute normalization for the observed $t\bar{t}$ distribution. In future experimental analyses, this normalization will undoubtedly be fitted to QCD at low transverse momenta where any effects from new physics are expected to be small. We need to take this renormalization into account in our theoretical analysis. Therefore, we rescale by a multiplicative constant the effective theory differential cross section which corresponds to the distribution that will be experimentally measured. We choose the constant so that the renormalized $d\sigma_{EFT}/dp_\perp$ cross section coincides with $d\sigma_{QCD}/dp_\perp$ at its maximum point.

One simple choice for a dimensionless measure of the difference between the QCD and EFT predictions for $d\sigma(PP \to t\bar{t})/dp_\perp$ is the ratio of their integrals:

$$R_{p_\perp} = \frac{\int dp_\perp (d\sigma_{EFT}/dp_\perp)}{\int dp_\perp (d\sigma_{QCD}/dp_\perp)}. \tag{4.1}$$

Since the disparity between $d\sigma_{QCD}/dp_\perp$ and $d\sigma_{EFT}/dp_\perp$ increases with $p_\perp$, we perform the integrations in the numerator and denominator of (4.1) only over the high transverse momentum range 500 GeV $\le p_\perp \le$ 1000 GeV in order to enhance the deviation of $R_{p_\perp}$ from unity. The dependence of $R_{p_\perp}$ upon the coefficient of the chromomagnetic moment operator $O_0^{(6)}$ is then much weaker than that for gluonic operators $O_1^{(6)}$ and $O_2^{(6)}$. So we plot $R_{p_\perp}$ as a function of $C_1^{(6)}(\Lambda)$ and $C_2^{(6)}(\Lambda)$ in fig. 6 with all other operator coefficients set equal to zero at the scale $\Lambda$.

The origin in fig. 6 necessarily lies along the $R_{p_\perp} = 1$ contour, for the effective field theory reduces to QCD at this point. It is clearly offset from the center of the concentric contours displayed in the figure. The offset is produced by the $O(1/\Lambda^2)$ interference terms in the squared amplitude expressions (3.1) and (3.3) which are linear in $C_1^{(6)}$ and $C_2^{(6)}$. The smallness of the displacement in the $C_1^{(6)}$ direction demonstrates that the term proportional to ${C_1^{(6)}}^2/\Lambda^4$ in $|A(gg \to t\bar{t})|^2$ dominates over the $C_1^{(6)}/\Lambda^2$ interference term as we have previously discussed.

While the ratio $R_{p_\perp}$ provides a useful global measure of the difference between the QCD and EFT transverse momentum $t\bar{t}$ distributions, two points in the $C_1^{(6)}$-$C_2^{(6)}$ plane that lie along the same contour in fig. 6 may correspond to two very different $d\sigma(PP \to t\bar{t})/dp_\perp$ curves. It is therefore instructive to consider a second dimensionless measure which is sensitive to the curves' shapes. To construct such a quantity, we first discretize the transverse momentum interval $500 \text{ GeV} \le p_\perp \le 1000$ GeV into $N = 20$ bins. We then multiply the value for $d\sigma/dp_\perp$ in each bin by the binwidth $\Delta p_\perp$, the integrated luminosity $\int \mathcal{L} dt$, the branching ratio BR for the $t\bar{t}$ pair's single lepton plus jets decay mode, and the $b$-tagging efficiency $\epsilon_b$ to convert the differential cross section into a corresponding number of observable $t\bar{t}$ events:

$$N_i = \left(\frac{d\sigma}{dp_\perp}\right)_i \times \Delta p_\perp \times \int \mathcal{L} dt \times \text{BR} \times \epsilon_b. \tag{4.2}$$

We shall take the numerical values for these parameters to be $\Delta p_\perp = 25$ GeV, $\int \mathcal{L} dt = 30 \text{ fb}^{-1}$, $\text{BR} = 24/81$ and $\epsilon_b = 0.25$. Finally, we quantify the difference between the effective theory distribution which will be measured for nonvanishing coefficient values and the theoretical QCD prediction in terms of a $\chi^2$ function. The highest $p_\perp$ bins are the most important for discriminating between the two distributions, but they contain the fewest events. So we adopt the Poisson $\chi^2$ function

$$\chi^2 = 2 \sum_{i=1}^{N} \Big[ N_i^{QCD} - N_i^{\text{obs}} + N_i^{\text{obs}} \ln \frac{N_i^{\text{obs}}}{N_i^{QCD}} \Big] \tag{4.3}$$

which is appropriate for low statistics [23].

We plot $\chi^2/N$ in fig. 7 as a function of $C_1^{(6)}(\Lambda)$ and $C_2^{(6)}(\Lambda)$ over the same region of coefficient parameter space as in fig. 6. The innermost crescent contour in the figure corresponds to the expectation value $\chi^2/N = 1$. For points lying within this contour, the probability that an observed transverse momentum differential cross section could be attributed to a statistical fluctuation of QCD rather than to nonvanishing values for the $C_1^{(6)}(\Lambda)$ and $C_2^{(6)}(\Lambda)$ coefficients is greater than 50 %. As QCD and the strong interaction effective theory cannot be meaningfully told apart inside the first contour, its boundary establishes a limit on the sensitivity to new gluon sector physics which can be achieved at the LHC. The surrounding contours in fig. 7 illustrate selected $\chi^2/N$ standard deviation levels where $\sigma = \sqrt{2/N} = 0.316$ for $N = 20$ degrees of freedom. For example, the gluonic operator coefficients lying on the outermost contour yield $d\sigma(PP \to t\bar{t})_{EFT}/dp_\perp$ distributions which can be distinguished from $d\sigma(PP \to t\bar{t})_{QCD}/dp_\perp$ at the $8\sigma$ level. If the horizontal and vertical axes were drawn to the same scale in fig. 7, this last contour would appear as an ellipse approximately three times more narrow in the $C_1^{(6)}(\Lambda)$ direction than in the $C_2^{(6)}(\Lambda)$ direction. The $\chi^2/N$ curves thus quantify the extent to which future LHC experiments will be more sensitive to the gluonic $G^3$ operator than to its four-quark competitors.

The information contained within the contour plots of fig. 6 and fig. 7 is insufficient to completely determine where an observed $d\sigma(PP \to t\bar{t})/dp_\perp$ function lies within the $C_1^{(6)}$-$C_2^{(6)}$ parameter space. But taken together, the two graphs significantly restrict the allowed values for these coefficients. Clearly, other differences between QCD and the effective field theory can be investigated along the lines which we have followed here. In particular, their predictions for the angular distributions of $t\bar{t}$ pairs can further constrain the allowed region within the coefficient parameter space. We briefly touch on this topic in the next section.

## 5. Top-antitop angular distributions

Our study of the prospects for detecting new strong interaction physics at the LHC has so far utilized only the transverse momentum information incorporated within $d^3\sigma(PP \to t\bar{t})/dy_3 dy_4 dp_\perp$. The triply differential cross section contains, however, complementary angular distribution information. We consider its implications for discriminating between the nonrenormalizable operators within the effective Lagrangian in this section.

A number of angular distributions for $t\bar{t}$ pairs as well as their decay products can be generated by integrating $d^3\sigma/dy_3 dy_4 dp_\perp$ over various ranges of $y_3$, $y_4$ and $p_\perp$. One differential cross section of particular interest is $d\sigma(PP \to t\bar{t})/d\cos\theta^*$ where $\theta^*$ denotes the angle between the direction of the boost and that of the top quark in the parton center-of-mass frame. We plot this cross section in fig. 8 for pure QCD and QCD plus some of the nonrenormalizable operators in our basis set. In order to enhance the operators' signal over the QCD background, we have imposed the transverse momentum cut $p_\perp \geq 500$ GeV. We have also required the lab frame angle between the $t$ or $\bar{t}$ and the beamline to exceed $25.4°$ to approximate the acceptance of planned LHC detectors. This last restriction ensures that the pseudorapidities of the decay products from high momentum tops will predominantly lie within the interval $-2.5 \leq \eta \leq 2.5$.

The solid curve in fig. 8 represents the QCD differential cross section $d\sigma(PP \to t\bar{t})_{QCD}/d\cos\theta^*$. The dotted, dashed and dot-dashed curves in the figure illustrate $d\sigma(PP \to t\bar{t})_{EFT}/d\cos\theta^*$ for $C_1^{(6)}(\Lambda) = 0.5$, $C_3^{(8)}(\Lambda) = 0.5$, and $C_3^{(8)}(\Lambda) = -0.5$ with $\Lambda = 2$ TeV. [7] We again find that the effect of the triple gluon field strength operator $O_1^{(6)}$ is essentially independent of the sign of its coefficient. The differential cross section corresponding to $C_1^{(6)}(\Lambda) = -0.5$ thus closely traces that for $C^{(6)}(\Lambda) = 0.5$ displayed in the figure. The dimension-8 gluon operator $O_3^{(8)}$ induces deviations from pure QCD which are clearly visible in $d\sigma/d\cos\theta^*$. This result is both surprising and interesting since we previously found that the effect of $O_3^{(8)}$ upon the $t\bar{t}$ transverse momentum distribution was negligible. Indeed, we did not include a curve corresponding to the dimension-8 gluon operator in the transverse momentum plots of fig. 4 and fig. 5 since it would have been suppressed relative to its dimension-6 counterpart by more than an order of magnitude. But we now see that experiments at the LHC can be sensitive to this next-to-next-to-leading order operator if they probe its impact on the $t\bar{t}$ angular distribution.

As for the transverse momentum distributions, it is again useful to identify dimensionless measures of the differences between the QCD and EFT predictions for $d\sigma(PP \to t\bar{t})/d\cos\theta^*$. One simple choice for such a measure is the ratio of their integrals

$$R_{\rm ang} = \frac{\int d\cos\theta^* (d\sigma_{EFT}/d\cos\theta^*)}{\int d\cos\theta^* (d\sigma_{QCD}/d\cos\theta^*)} \tag{5.1}$$

which is the analog of $R_{p_\perp}$ in eqn. (4.1). Another is the ratio $R_{\rm rms}$ of their root-mean-squared values for $\cos\theta^*$. We have calculated these ratios for the angular differential cross sections shown in fig. 8 and for similar cross sections involving the magnetic moment and four-quark operators. For the curves corresponding to $C_0^{(6)}(\Lambda)$, $C_1^{(6)}(\Lambda)$, $C_2^{(6)}(\Lambda)$ and $C_3^{(8)}(\Lambda)$ equal to 0.5, we find $R_{\rm ang} = (1.03, 1.23, 1.46, 0.82)$ and $R_{\rm rms} = (0.999, 0.978, 0.991, 0.871)$. For analogous curves with the $\Lambda$ scale coefficients set equal to -0.5, we find $R_{\rm ang} = (0.969, 1.23, 0.735, 1.18)$ and $R_{\rm rms} = (1.00, 0.978, 1.03, 1.08)$.

[7] The coefficient $C_3^{(8)}$ was not evolved using the renormalization group but was instead simply fixed at its $\Lambda$ scale value.

The most striking conclusion which we draw from these results is that the dimension-8 gluon operator alters the shape of the $t\bar{t}$ angular distribution much more than all the other dimension-6 operators in $\mathcal{L}_{\rm eff}$ for comparable values of their coefficients. The magnetic moment operator's angular distribution is indistinguishable from that of pure QCD, while the distributions of the $G^3$ and $(DG)^2$ operators differ significantly from that of QCD in $R_{\rm ang}$ but not in $R_{\rm rms}$. $O_3^{(8)}$ thus possesses a distinctive signature: a QCD-like transverse momentum distribution but a quite nonstandard angular distribution with $R_{\rm ang}$ and $R_{\rm rms}$ both deviating from unity in the same direction.

A more precise determination of the power of angular measurements to delimit allowed regions of coefficient parameter space will require detailed simulations including top quark decays and detector resolution. We leave such a study to future work.

## 6. Conclusions

In this article, we have investigated the impact of the triple gluon field strength operator upon top quark pair production. The $G^3$ operator represents the only genuinely gluonic CP even term which can arise at dimension-6 within an effective strong interaction Lagrangian. Although it has proven surprisingly difficult to study in the past, the prospects for either detecting or significantly constraining this operator appear quite promising at the LHC where its effects will be significantly enhanced by the large gluon content at small $x$ of the colliding hadrons. We have found that the sensitivity of the $t\bar{t}$ transverse momentum distribution to the triple field strength operator is greater than that to all other dimension-6 pure quark and mixed quark-gluon terms in $\mathcal{L}_{\rm eff}$ for comparable values of their coefficients. Moreover, only one higher order gluonic operator can contribute to $gg \to t\bar{t}$ at dimension-8, and its effect on the $p_\perp$ distribution is more than an order of magnitude smaller than that of $G^3$. We have also seen that angular distribution information can help to differentiate effects from the various operators which may reside within the effective Lagrangian. Top-antitop production therefore promises to provide an important means for probing new strong interaction physics beyond the Standard Model.


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## Figure Captions

Fig. 1. Diagrams with a heavy colored fermion or boson running around the loop which match onto gluonic operators in the effective strong interaction Lagrangian.

Fig. 2. (a) Lowest order QCD graphs which contribute to $gg \to t\bar{t}$ scattering. (b) $O(1/\Lambda^2)$ $gg \to t\bar{t}$ graphs with single insertions of the chromomagnetic moment operator $O_0^{(6)}$. (c) $O(1/\Lambda^2)$ $gg \to t\bar{t}$ graph with single insertion of the triple gluon field strength operator $O_1^{(6)}$. (d) $O(1/\Lambda^4)$ $gg \to t\bar{t}$ graphs with two insertions of either $O_0^{(6)}$ or $O_0^{(6)}$ and $O_1^{(6)}$.

Fig. 3. Lowest order QCD, chromomagnetic moment and four-quark operator graphs which contribute to $q\bar{q} \to t\bar{t}$ scattering.

Fig. 4. Transverse momentum differential cross section $d\sigma(PP \to t\bar{t})/dp_\perp$ calculated for an LHC center-of-mass energy $\sqrt{s} = 14$ TeV. The solid curve illustrates the pure QCD cross section. The dot-dashed, dashed and dotted curves represent the additional nonrenormalizable operator contributions obtained after respectively setting $C_0^{(6)}(\Lambda)$, $C_1^{(6)}(\Lambda)$ and $C_2^{(6)}(\Lambda)$ equal to 0.5 with $\Lambda = 2$ TeV.

Fig. 5. Transverse momentum differential cross section $d\sigma(P\overline{P} \to t\bar{t})/dp_\perp$ calculated for a Tevatron center-of-mass energy $\sqrt{s} = 1.8$ TeV. The curves are labeled the same as those in fig. 4. The dashed $O_1^{(6)}$ curve must be subtracted from rather than added to the solid QCD curve to obtain the EFT cross section that corresponds to $C_1^{(6)}(\Lambda) = 0.5$.

Fig. 6. Ratio $R_{p_\perp}$ of the EFT and QCD $t\bar{t}$ transverse momentum distributions integrated over 500 GeV $\le p_\perp \le$ 1000 GeV plotted as a function of gluonic operator coefficients $C_1^{(6)}(\Lambda)$ and $C_2^{(6)}(\Lambda)$ with $\Lambda = 2$ TeV and $\sqrt{s} = 14$ TeV. The values for $R_{p_\perp}$ are displayed alongside the contours.

Fig. 7. $\chi^2/N$ for $N = 20$ transverse momentum bins plotted as a function of gluonic operator coefficients $C_1^{(6)}(\Lambda)$ and $C_2^{(6)}(\Lambda)$ with $\Lambda = 2$ TeV and $\sqrt{s} = 14$ TeV. The innermost crescent contour corresponds to $\chi^2/N = 1$. The surrounding contours represent $\chi^2/N = 1 + 2n\sigma$ where $\sigma = \sqrt{2/N} = 0.316$ and $n = 1, 2, 3, 4$.

Fig. 8. Angular differential cross section $d\sigma(PP \to t\bar{t})/d\cos\theta^*$ calculated for $\sqrt{s} = 14$ TeV. The solid curve illustrates the pure QCD cross section. The dotted curve corresponds to QCD plus the dimension-6 operator $O_1^{(6)}$ with $C_1^{(6)}(\Lambda) = 0.5$ and $\Lambda = 2$ TeV. The dashed and dot-dashed curves represent QCD plus the dimension-8 operator $O_3^{(8)}$ with $C_3^{(8)}(\Lambda)$ set equal to 0.5 and -0.5 respectively.